4-19

Trig Integrals

$$\int \tan x\, dx = \int \frac{\sin x}{\cos x}\, dx \qquad u = \cos x, \quad du = -\sin x\, dx$$

$$= -\ln|\cos x| + C$$

$$\int \cot x\, dx = \int \frac{\cos x}{\sin x}\, dx \qquad u = \sin x, \quad du = \cos x$$

$$= \ln|\sin x| + C$$

$$\int \sec x\, dx = \int \sec x \frac{(\tan x + \sec x)}{(\sec x + \tan x)}\, dx \qquad u = \sec x + \tan x, \quad du = \sec x \tan x + \sec^2 x\, dx$$

$$= \ln|\sec x + \tan x| + C$$

$$\int \csc x\, dx = \int \csc x \frac{(\csc x + \cot x)}{(\cot x + \csc x)}\, dx \qquad u = \cot x + \csc x, \quad du = -\csc^2 x - \csc x \cot x\, dx$$

$$= -\ln|\cot x + \csc x| + C$$

① $\int \cot \frac{\theta}{3}\, d\theta$

$u = \frac{\theta}{3}$

$du = \frac{1}{3}\, d\theta$

$$= 3 \int \frac{1}{3} \cot\left(\frac{\theta}{3}\right) d\theta$$

$$= 3 \int \cot u\, du$$

$$= \boxed{3 \ln\left|\sin \frac{\theta}{3}\right| + C}$$

② $\int \csc(2x)\, dx$

$u = 2x$

$du = 2\, dx$

$$= \frac{1}{2} \int \csc u\, du$$

$$= -\frac{1}{2} \ln|\cot u + \csc u| + C$$

$$= \boxed{-\frac{1}{2} \ln|\cot 2x + \csc 2x| + C}$$

③ $\int \cos 3\theta - 1 \, d\theta$

$= \int \cos 3\theta \, d\theta - \int d\theta$

$u = 3\theta$ no u sub

$du = 3\, d\theta$

$= \frac{1}{3} \int \cos u \, du - \int 1 \, d\theta$

$= \frac{1}{3} \sin u - \theta + C$

$= \boxed{\frac{1}{3} \sin(3\theta) - \theta + C}$

④ $\int \frac{\cos t}{1 + \sin t} \, dt$

$u = 1 + \sin t$

$du = \cos t \, du$

$= \int \frac{du}{u}$

$= \ln|u| + C$

$= \boxed{\ln|1 + \sin t| + C}$

⑤ $\int \frac{\sec x \tan x}{\sec x - 1} \, dx$

$u = \sec x - 1$

$du = \sec x \tan x \, dx$

$= \int \frac{du}{u}$

$= \ln|u| + C$

$= \boxed{\ln|\sec x - 1| + C}$

⑥ $\int \sec 2x + \tan 2x \, dx$

$= \int \sec 2x \, dx + \int \tan 2x \, dx$

$u = 2x$

$du = 2\, dx$

$= \frac{1}{2} \int \sec u \, du + \frac{1}{2} \int \tan u \, du$

$= \frac{1}{2} \ln|\sec u + \tan u|$

$- \frac{1}{2} \ln|\cos u| + C$

$= \boxed{\frac{1}{2} \ln|\sec 2x + \tan 2x| - \frac{1}{2} \ln|\cos 2x| + C}$

error in answers

⑦ $\int \tan 5\theta \, d\theta$

$u = 5\theta$

$du = 5\, d\theta$

$= \frac{1}{5} \int \tan u \, du$

$= -\frac{1}{5} \ln|\cos u| + C$

$= \boxed{-\frac{1}{5} \ln|\cos 5\theta| + C}$

(8) $\int \sec \frac{x}{2}\, dx$

$u = \frac{x}{2}$

$du = \frac{1}{2} dx$

$= 2\int \sec u \, du$

$= 2 \ln|\sec u + \tan u| + C$

$= \boxed{2 \ln\left|\sec \frac{x}{2} + \tan \frac{x}{2}\right| + C}$

(9) $\int 2 - \tan \frac{\theta}{4}\, d\theta$

$= \int 2\, d\theta - \int \tan \frac{\theta}{4}\, d\theta$

no u-sub

$u = \frac{\theta}{4}$

$du = \frac{1}{4} d\theta$

$= 2\theta + C - 4\int \tan u\, du$

$= 2\theta + C + 4 \ln|\cos u|$

$= \boxed{2\theta + 4 \ln\left|\cos \frac{\theta}{4}\right| + C}$

(10) $\int \frac{\csc^2 t}{\cot t}\, dt$

$u = \cot t$

$du = -\csc^2 t\, dt$

$= -\int \frac{du}{u}$

$= -\ln|u| + C$

$= \boxed{-\ln|\cot t| + C}$

(11) $\int_1^2 \frac{1-\cos\theta}{\theta - \sin\theta}\, d\theta$

$u = \theta - \sin\theta$

$du = 1 - \cos\theta\, d\theta$

$u_1 = 1 - \sin 1$

$u_2 = 2 - \sin 2$

$= \int_{u=1-\sin 1}^{u=2-\sin 2} \frac{du}{u}$

$= \ln|u| \Big|_{u=1-\sin 1}^{u=2-\sin 2}$

$= \ln|\theta - \sin\theta| \Big|_{\theta=1}^{\theta=2}$

$= \ln|2-\sin 2| - \ln|1-\sin 1|$

$= \boxed{\ln\left|\frac{2-\sin 2}{1-\sin 1}\right|}$ log property!

(12) $\int_{0.1}^{0.2} (\csc 2\theta - \cot 2\theta)^2\, d\theta$

FOIL first

$= \int_{0.1}^{0.2} \underbrace{\csc^2 2\theta}_{\text{known}} - \underbrace{2\csc 2\theta \cot 2\theta}_{\text{known}} + \underbrace{\cot^2 2\theta}_{\text{uh-oh... trig identity}}\, d\theta$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$ Pythagorean identity

$$1 + \cot^2 x = \csc^2 x$$

$$\cot^2 x = \csc^2 x - 1$$

So ...

$$\cot^2 2\theta = \csc^2 2\theta - 1$$

$$\text{Integral} = \int_{0.1}^{0.2} \csc^2 2\theta - 2\csc 2\theta \cot 2\theta + \csc^2 2\theta - 1 \, d\theta$$

combine like terms

$$= \int_{0.1}^{0.2} 2\csc^2 2\theta - 2\csc 2\theta \cot 2\theta - 1 \, d\theta$$

$$= \int_{0.1}^{0.2} 2\csc^2 2\theta \, d\theta - \int_{0.1}^{0.2} 2\csc 2\theta \cot 2\theta - \int_{0.1}^{0.2} d\theta$$

$u = 2\theta$ same   no u-sub

$du = 2\,d\theta$

$u_1 = 2(0.1) = 0.2$

$u_2 = 2(0.2) = 0.4$

$$= \int_{u=0.2}^{u=0.4} \csc^2 u \, du - \int_{u=0.2}^{u=0.4} \csc u \cot u \, du - \theta \Big|_{\theta=0.1}^{\theta=0.2}$$

$$= -\cot u \Big|_{u=0.2}^{u=0.4} + \csc u \Big|_{u=0.2}^{u=0.4} - (0.2 - 0.1)$$

$$= -\cot 2\theta \Big|_{\theta=0.1}^{\theta=0.2} + \csc 2\theta \Big|_{\theta=0.1}^{\theta=0.2} - 0.1$$

$$= \boxed{-\cot(0.4) + \cot(0.2) + \csc(0.4) - \csc(0.2) - 0.1}$$

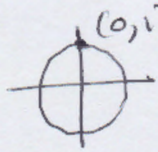
answer did not combine like terms

(13) $\displaystyle\int_{\pi/4}^{\pi/2} \csc x - \sin x \, dx$

$$= \int_{\pi/4}^{\pi/2} \csc x \, dx - \int_{\pi/4}^{\pi/2} \sin x \, dx$$

$$= -\ln\left|\cot x + \csc x\right| \Big|_{x=\pi/4}^{x=\pi/2} + \cos x \Big|_{x=\pi/4}^{x=\pi/2}$$

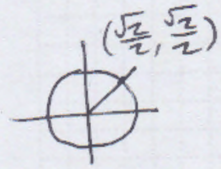
$$= -\ln\left|\cot\frac{\pi}{2} + \csc\frac{\pi}{2}\right| + \ln\left|\cot\frac{\pi}{4} + \csc\frac{\pi}{4}\right| + \cos\frac{\pi}{2} - \cos\frac{\pi}{4}$$



$\cos\frac{\pi}{2} = 0$

$\cot\frac{\pi}{2} = 0$

$\csc\frac{\pi}{2} = 1$



$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cot\frac{\pi}{4} = 1$

$\csc\frac{\pi}{4} = \sqrt{2}$

$$= -\ln\left|0+1\right| + \ln\left|1+\sqrt{2}\right| + 0 - \frac{\sqrt{2}}{2}$$

$1+\sqrt{2} > 0$
no abs value

$$= -\ln(1) + \ln(1+\sqrt{2}) - \frac{\sqrt{2}}{2}$$

$$= \boxed{\ln(1+\sqrt{2}) - \frac{\sqrt{2}}{2}}$$

(14) $\displaystyle\int_{-\pi/4}^{\pi/4} \frac{\sin x - \cos^3 x}{\cos^2 x} \, dx$

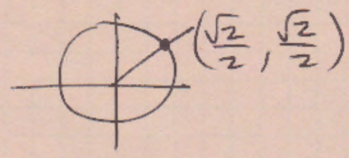
$$= \int_{-\pi/4}^{\pi/4} \frac{\sin x}{\cos^2 x} \, dx - \int_{-\pi/4}^{\pi/4} \frac{\cos^3 x}{\cos^2 x} \, dx$$

$$= \int_{-\pi/4}^{\pi/4} \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \, dx - \int_{-\pi/4}^{\pi/4} \cos x \, dx$$

$$= \int_{-\pi/4}^{\pi/4} \tan x \sec x \, dx - \int_{-\pi/4}^{\pi/4} \cos x \, dx$$

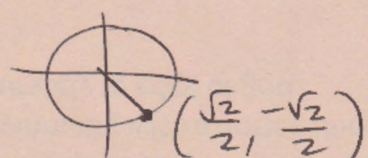
$$= (\sec x - \sin x) \Big|_{-\pi/4}^{\pi/4}$$

$$= \left(\sec\frac{\pi}{4} - \sin\frac{\pi}{4}\right) - \left(\sec\left(-\frac{\pi}{4}\right) - \sin\left(-\frac{\pi}{4}\right)\right)$$



$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\sec\frac{\pi}{4} = \sqrt{2}$



$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\sec\left(-\frac{\pi}{4}\right) = \sqrt{2}$

$$= \sqrt{2} - \frac{\sqrt{2}}{2} - \left(\sqrt{2} - \left(-\frac{\sqrt{2}}{2}\right)\right)$$

$$= \sqrt{2} - \frac{\sqrt{2}}{2} - \sqrt{2} - \frac{\sqrt{2}}{2}$$

$$= -2\frac{\sqrt{2}}{2}$$

$$= \boxed{-\sqrt{2}}$$